

Course-M.Sc. (Mathematics)

Session- May-2017

Subject –Advance Algebra

Subject Code- MMAT1-205

Sem-2<sup>nd</sup>

Date of submission- 17/02/17

Assignment –I (Unit-I)

Q-1 Every Inner Product Space is a Metric Space.

Q-2 Let  $V$  be an Inner Product Space. Then  $|(u, v)| \leq \|u\| \|v\|$  for all  $u, v \in V$

Q-3 If  $\{w_1, w_2, w_3, \dots, w_m\}$  is an orthonormal set in  $V$ . then  $\sum_{i=1}^m |w_i, v|^2 \leq \|v\|^2$  for all  $v \in V$ .

Q-4 State and Prove Gram-Schmidt Orthogonalisation process.

Q-5 Let  $W$  be the subspace of  $\mathbb{R}^5$  spanned by the vectors  $a = (2, 2, 3, 4, -1)$ ,

$b = (-1, 1, 2, 5, 2)$ ,  $c = (0, 0, -1, -2, 3)$ ,  $d = (1, -1, 2, 3, 0)$ . Describe  $A(W)$ .

Q-6 Let  $V$  be the vector space of all Polynomial functions  $P$  from  $\mathbb{R}$  into  $\mathbb{R}$  which have degree 2

or less. Define three linear functional on  $V$  by  $f(x) = \int_0^1 p(x) dx$ ,  $g(x) = \int_0^2 p(x) dx$ ,

$h(x) = \int_0^{-1} p(x) dx$  show that  $\{f, g, h\}$  is basis of  $V$ . Determine a basis for  $V$  where  $\{f, g, h\}$

is its dual basis.

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Assignment –II (Unit-II)

Q-1 State and Prove Eienstein Criterion and  $f(x) = x^2 - 6x + 12$  is irreducible or not over  $\mathbb{Z}$ .

Q-2 Find degree and basis of field extension  $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$  and  $\mathbb{Q}(\sqrt{2}, \sqrt{3})$  over  $\mathbb{Q}$ .

Q-3 If  $F \subset E \subset K$  and  $[E:F] < \infty$  &  $[K:E] < \infty$  Then (a)  $[K:F] < \infty$ ,

(b)  $[K:F] = [K:E][E:F]$  (c)  $[K:E]/[K:F] \& [E:F]/[K:F]$

Q-4 Let  $p(x)$  be an irreducible polynomial in  $F[x]$  and  $u$  be a root of  $p(x)$  in some field extension  $K$  of  $F$ . Then

(a)  $F[u] = \{b_0 + b_1u + b_2u^2 + \dots + b_mu^m / b_0 + b_1x + b_2x^2 + \dots + b_mx^m \in F[x]\}$  i.e  $F(u) = F[u]$ .

(b) If degree of  $p(x) = n$ , then  $\{1, u, u^2, \dots, u^{n-1}\}$  forms basis of  $F(u)$  over  $F$ .

Q-5 If  $E$  is finite extension of  $F$ . Then  $E$  is algebraic over  $F$ . Is the converse true?

Q-6 For any field  $K$  the following are equivalent.

(a)  $K$  is algebraically closed.

(b) Every irreducible polynomial in  $k[x]$  is of deg 1.

(c) Every polynomial in  $K[x]$  of positive degree factorize completely in  $K[x]$  into linear factors.